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| Created on: | July, 2015 |
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| Revised on: |  |
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| **OCEAN COUNTY MATHEMATICS CURRICULUM** | | | | | |
| **Content Area:** Mathematics | | | | | |
| **Course Title:** Calculus I | | | | **Grade Level:**  High School | |
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|  | **Pre-Requisite Skills** |  | **20 Days** | |  |
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|  | **Overview of Calculus** |  | **5 Days** | |  |
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|  | **Limits and Continuity** |  | **10 Days** | |  |
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|  | **The Derivative** |  | **30 Days** | |  |
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|  | **Applications of the Derivative** |  | **25 Days** | |  |
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|  | **Higher Order of Derivatives and Graphing Implications** |  | **15 Days** | |  |
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|  | **Anti-Derivatives** and **The Definite Integral** |  | **26 Days** |  | |
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|  | **Differential Equations** |  | **22 Days** |  | |
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|  | **Applications of Integration** |  | **24 Days** |  | |
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The following Standards for Mathematical Practice and select Common Core Content Standards should be covered throughout the various units of the curriculum.

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| Standards for Mathematical Practices | | |
| MP.1 | Make sense of problems and persevere in solving them. | * Find meaning in problems * Look for entry points * Analyze, conjecture and plan solution pathways * Monitor and adjust * Verify answers * Ask themselves the question: “Does this make sense?” |
| MP.2 | Reason abstractly and quantitatively. | * Make sense of quantities and their relationships in problems * Learn to contextualize and decontextualize * Create coherent representations of problems |
| MP.3 | Construct viable arguments and critique the reasoning of others. | * Understand and use information to construct arguments * Make and explore the truth of conjectures * Recognize and use counterexamples * Justify conclusions and respond to arguments of others |
| MP.4 | Model with Mathematics. | * Apply mathematics to problems in everyday life * Make assumptions and approximations * Identify quantities in a practical situation * Interpret results in the context of the situation and reflect on whether the results make sense |
| MP.5 | Use appropriate tools strategically. | * Consider the available tools when solving problems * Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools) * Make sound decisions of which of these tools might be helpful |
| MP.6 | Attend to precision. | * Communicate precisely to others * Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes * Calculate accurately and efficiently |
| MP.7 | Look for and make use of structure. | * Discern patterns and structures * Can step back for an overview and shift perspective * See complicated things as single objects or as being composed of several objects |
| MP.8 | Look for and express regularity in repeated reasoning. | * Notice if calculations are repeated and look both for general methods and shortcuts * In solving problems, maintain oversight of the process while attending to detail * Evaluate the reasonableness of their immediate results |

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| Technology goals for Calculus: |
| Students will be able to use a graphing calculator to graph a function, find the zeros of a function, find the extrema of a function, analyze the graph to determine domain and range, analyze and interpret tables of data, create scatter plots and use the regression feature including calculating the correlation coefficient, solve a system by finding the point of intersection, evaluate a logarithm, evaluate an exponential expression, evaluate a trigonometric function, evaluate an inverse trigonometric function, find the numerical value of a derivative, evaluate a definite integral. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area:** Mathematics  **Grade:** High School | | |
| **Unit Title:** Pre-Requisite Skills | | |
| **Domain:** High School: Algebra, Functions, Modeling, & Geometry | | |
| **Unit Summary:**  This unit will review pre-requisite skills necessary for success in calculus. The skills include solving equations and inequalities, graphing functions and relations, simplifying, graphing, adding, subtracting and multiplying rational expressions; understanding and using function notation; evaluating and simplifying trigonometric expressions; solving and graphing trig equations.  **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology. | | |
| **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
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| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| A-APR.6 | Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. | |
| A-CED.1 | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | |
| A-CED.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | |
| A-REI.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |
| A-REI.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | |
| A-REI.4 | Solve quadratic equations in one variable. | |
| A-REI.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | |
| F-IF.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). | |
| F-IF.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | |
| F-IF.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. | |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. | |
| F-IF.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | |
| F-IF.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | |
| F-IF.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | |
| F-IF.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. | |
| F-BF.1 | Write a function that describes a relationship between two quantities. | |
| F-BF.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | |
| F-BF.3 | Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. | |
| F-BF.4 | Find inverse functions. | |
| F-BF.5 | Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | |
| F-LE.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |
| F-TF.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | |
| F-TF.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | |
| G-MG.1. | Use geometric shapes, their measures, and their properties to describe objects. | |
| G-MG.3. | Apply geometric methods to solve design problems. | |
| **Unit Essential Questions**   * Are the necessary pre-requisite skills in place for success in calculus? | | **Unit Enduring Understandings**  *Students will understand that…*   * There are certain skills from previous math courses that are essential to success in Calculus. |
| **Unit Objectives**  *Students will know…*   * How to solve equations and inequalities. * How to graph relations and functions. * How to simplify, graph, add, subtract, multiply, and divide rational expressions. * How to understand and use function notation. * How to simplify and evaluate trigonometric expressions. * How to solve and graph trigonometric equations. | | **Unit Objectives**  *Students will be able to…*   * Solve equations and inequalities * Graph relations and functions * Simplify, graph, add, subtract, multiply, and divide rational expressions. * Understand and use function notation. * Simplify and evaluate trigonometric expressions. * Solve and graph trigonometric equations. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | |
| **Formative Assessments** | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share | * DO-NOW * Notebook * Writing prompts * Exit passes * Self-assessment |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes * Presentations * Unit Projects * Mid-Term and Final Exams | |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | |
| **Teacher Notes:** | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area: Mathematics Grade: High School** | | |
| **Unit Title:** Overview of Calculus | | |
| **Domain:** Number & Quantity/Algebra/Functions/Modeling/Geometry | | |
| **Unit Summary:**  The students will be introduced to the main concepts in Calculus. The unit is based on the Paul Forrester Calculus: Concepts and Applications textbook from Key Curriculum Press. The students get a brief overview of the concepts of limits, instantaneous rate of change and area under a curve via rectangles and trapezoids.  **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology. | | |
| **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
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| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| N-RN.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | |
| N-Q.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | |
| N-Q.2 | Define appropriate quantities for the purpose of descriptive modeling. | |
| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |
| A-CED.1 | Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | |
| A-CED.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | |
| **Number** | **Common Core Standard for Mastery** | |
| A-REI.11 | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ | |
| F-IF.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the* *function is increasing, decreasing, positive, or negative; relative maximums* *and minimums; symmetries; end behavior; and periodicity.* | |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* *h(n) gives the number of person-hours it takes to assemble n engines in a* *factory, then the positive integers would be an appropriate domain for the* *function.* | |
| F-IF.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | |
| F-BF.1 | Write a function that describes a relationship between two quantities. | |
| F-LE.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | |
| F-LE.4 | For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. | |
| FL-E.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |
| G-MG.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ | |
| **Unit Essential Questions**   * How much has a quantity changed? * How quickly does a quantity change? | | **Unit Enduring Understandings**  *Students will understand that…*   * A function can be interpreted graphically, analytically and numerically. * The rate of change of any function can be found at a particular instant. * The amount that a function has changed over a given time can be interpreted from a graph. |
| **Unit Objectives**  *Students will know…*   * The limit of a function at a point happens as the values of x get close to but not necessarily equal to x. * The derivative of a function represents the instantaneous rate of change. * The area under a curve is the amount of change of a function over time. | | **Unit Objectives**  *Students will be able to…*   * Evaluate a limit * Determine the instantaneous rate of change of a function * Use rectangles and trapezoids to approximate area under a curve. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | | |
| **Formative Assessments** | | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share | * DO-NOW * Notebook * Writing prompts * Exit passes * Self-assessment | |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes | | * Presentations * Unit Projects * Mid-Term and Final Exams |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | | |
| **Teacher Notes:** | | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area: Mathematics Grade:** High School | | |
| **Domain:** Number & Quantity/Algebra/Functions/ Modeling, Geometry | | |
| **Unit**: Limits | | |
| **Unit Summary**  Students to connect PreCalculus to Calculus through the limit process. Geometrical, numerical, and algebraic methods will be explored. Topics include: Continuous - discontinuous functions and the effect on finding limits (continuity), the definition of limit, the explanation of why limits fail, limit properties and how they make finding limits easier, the squeeze theorem and finding limits of trigonometric functions, IVT, EVT, Continuity and one sided limits, limits at infinity, and the epsilon delta definition of limits. | | |
| **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology.  **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
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| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement  when reporting quantities | |
| A-SSE.3 | Choose and produce an equivalent form of an expression to reveal and  explain properties of the quantity represented by the expression | |
| A-APR.6 | Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*)  in the form *q*(*x*) + *r*(*x*)/*b*(*x*), where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are  polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using  inspection, long division, or, for the more complicated examples, a  computer algebra system. | |
| F-IF.4 | For a function that models a relationship between two quantities,  interpret key features of graphs and tables in terms of the quantities,  and sketch graphs showing key features given a verbal description  of the relationship. | |
| F-IF.7 | Graph functions expressed symbolically and show key features of  the graph, by hand in simple cases and using technology for more  complicated cases | |
| **Unit Essential Questions**   * How do limits connect average rate of change to instantaneous rates of change? * What is the connection between a limits and continuity? | | **Unit Enduring Understandings**  *Students will understand that…*   * Without limits there is no Calculus. * Limits determine the behavior of a function as it gets closer to certain values |
| **Unit Objectives**  *Students will know…*   * The connection of pre-calculus to calculus through limits * Continuous and Discontinuous functions and how they relate to limits * The definition of limit * When limits fail to exist * How to use limit properties to evaluate limits * How the squeeze theorem is derived and its application to trigonometric limits * How continuity and limits are related * How continuity and one-sided limits are related * To find limits at infinity * Use the epsilon delta definition of limit to find delta for a given epsilon | | **Unit Objectives**  *Students will be able to…*   * Explain why the concept of limit was important in solving the tangent line and area problems * Explain and write in mathematical terms the definition of limit. * Show geometrically when a limit does not exist and give algebraic examples of those functions. * State the relationship between continuity and limits vs discontinuity and limits * Use graphs, numerical tables, and algebra methods to find limits * Create a strategy which includes limit properties and methods to evaluate various limits. * Explain the IVT and EVT theorems verbally and geometrically * State how one-side limits help evaluate functions such as piecewise functions, step functions, and rational functions * Find corresponding delta values for given epsilon values and can you relate this to real world applications |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | |
| **Formative Assessments** | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share |  |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes * Presentations * Unit Projects * Mid-Term and Final Exams | |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | |
| **Teacher Notes:** | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area:** Mathematics  **Grade:** High School | | |
| **Unit:** The Derivative | | |
| **Domain:** Number & Quantity/Algebra/Functions/Modeling/Geometry | | |
| **Unit Summary**  The derivative is a mathematical breakthrough from the 17th century. It helps to answer the question as to “how fast” certain quantities are changing. The derivative gives the slope of the tangent line to a function which measures the instantaneous rate of the function at a given point. The derivative can be calculated graphically, analytically or numerically.  **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology. | | |
| **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
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| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| N-RN.2 | Explain how the definition of the meaning of rational exponents  follows from extending the properties of integer exponents to  those values, allowing for a notation for radicals in terms of rational  exponents. | |
| N-Q.1 | Use units as a way to understand problems and to guide the solution  of multi-step problems; choose and interpret units consistently in  formulas; choose and interpret the scale and the origin in graphs and  data displays. | |
| N-Q.2 | Define appropriate quantities for the purpose of descriptive modeling. | |
| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement  when reporting quantities. | |
| A-SSE.1 | Interpret expressions that represent a quantity in terms of its context. | |
| A-SSE.3 | Choose and produce an equivalent form of an expression to reveal and  explain properties of the quantity represented by the expression. | |
| A-CED.1 | Create equations and inequalities in one variable and use them to  solve problems. Include equations arising from linear and quadratic  functions, and simple rational and exponential functions. | |
| A-CED.4 | Rearrange formulas to highlight a quantity of interest, using the same  reasoning as in solving equations. For example, rearrange Ohm’s law V =  IR to highlight resistance R. | |
| F-IF.4 | For a function that models a relationship between two quantities,  interpret key features of graphs and tables in terms of the quantities,  and sketch graphs showing key features given a verbal description  of the relationship. | |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to  the quantitative relationship it describes. | |
| F-IF.6 | Calculate and interpret the average rate of change of a function  (presented symbolically or as a table) over a specified interval.  Estimate the rate of change from a graph. | |
| F-BF.1 | Write a function that describes a relationship between two quantities. | |
| **Unit Essential Questions**   * Can we find the derivative of at any moment? * Does every function have a derivative? * Does a function have a rate of change everywhere? * Can the derivative be used to predict future values of the function? (tangent line approximations) * When and why would a derivative fail to exist. | | **Unit Enduring Understandings**  *Students will understand that…*   * Derivatives determine slope of a curve at any given point. * Derivative explains the rate at which quantities change. |
| **Unit Objectives**  *Students will know…*   * Derivatives of a function * Differentiability of a function * Rules for differentiation * Trigonometric derivatives * Implicit differentiation * Derivatives of Inverse Functions * Derivatives of Exponential and Logarithmic Functions | | **Unit Objectives**  *Students will be able to…*   * Find the first derivative and higher order derivatives of any explicit or implicit function. * Recognize the structure of the function and understand what rules apply and in what order to perform the rules. * Use the derivative to find a linear approximation. * Use the derivative to find rates of change for various application problems, including but not limited to physics and/or business. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | |
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| **Summative Assessments**   * Chapter/Unit Test * Quizzes * Presentations * Unit Projects * Mid-Term and Final Exams | |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | |
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| **Teacher Notes:** | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area:** Mathematics  **Grade:** High School | | |
| **Unit Title:** Applications of the Derivative | | |
| **Domain:**  Number and Quantity, Algebra, Functions, Modeling, Geometry | | |
| **Unit Summary:**  The students will be able to analyze and solve problems where the quantities involved change with respect to time (related rates problems), using implicit differentiation. They will also be able to use the rules of differentiation to analyze and solve optimization problems. The Mean Value Theorem, Rolle’s Theorem and the Intermediate Value Theorem for derivatives will be explored in real life situations and applied to the related rates and optimization problems.  **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology.  **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
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| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| N-Q.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays | |
| N-Q.2 | Define appropriate quantities for the purpose of descriptive modeling. | |
| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ | |
| A-CED.1 | Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | |
| A-CED.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | |
| F-TF.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ | |
| F-TF.7. | (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ | |
| F-TF.7 | (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ | |
| G-SRT.11 | (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | |
| G-MG.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ | |
| F-BF.1 | Write a function that describes a relationship between two quantities. | |
| F-LE.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions.   * + Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.   + Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.   + Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | |
| **Unit Essential Questions**   * How are Rolle’s Theorem and the Mean Value Theorem related? * Related rates problems describe real life situations. Can you identify the variable quantities in a problem and set up the relationships between the quantities as mathematical equations? * Can you represent the problem as   a picture, labeling all pertinent  information and solve the problem  presented?   * Optimization problems are used in   business applications as well as  physics applications. Can you  identify the variables and the  equations necessary to find the  maximum and minimum function  values?   * Do you know the relationship between the between supply and demand curves? Can you maximize profit and minimize cost? | | **Unit Enduring Understandings**  *Students will understand that…*   * The average rate of change is related to the   instantaneous rate of change (algebra vs. calculus).   * Implicit differentiation is necessary in solving   related rates problems, as more than one quantity  can change with respect to time.   * Calculus is the language of physics, and with calculus we can create mathematical models for certain physical and business situations. * Differentiation is the tool for solving problems involving optimization. |
| **Unit Objectives**  *Students will know…*   * Whether a function is expressed explicitly of implicitly. * How to find a derivative using implicit differentiation. * When to use the appropriate differentiation rules as they implicitly differentiate. * The various methods for finding extrema on both a closed interval and an open interval. * The relationship between position, velocity and acceleration. | | **Unit Objectives**  *Students will be able to…*   * Distinguish between functions written in explicit form and implicit form. * Find the derivative of a function expressed implicitly. * Use the rules for differentiation when differentiating implicitly. * Set up and solve related rates problems. * Locate extrema on a closed interval. * Determine if Rolle’s Theorem is applicable and if so, apply the theorem. * Determine if the Mean Value Theorem is applicable and if so, find the values guaranteed by the theorem. * Know the relationship between the position function, the velocity function and the acceleration function. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | |
| **Formative Assessments** | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share | * DO-NOW * Notebook * Writing prompts * Exit passes * Self-assessment |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes * Presentations * Unit Projects * Mid-Term and Final Exams | |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | |
| **Teacher Notes:** | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area:** Mathematics  **Grade:** High School | | |
| **Unit Title**: Higher Order of Derivatives and Graphing Implications | | |
| **Domain:** Number & Quantity/Algebra/Functions/Modeling/Geometry | | |
| **Unit Summary:**  This unit shows how to draw conclusions from derivatives about the extreme values of a function and about the general shape of a function’s graph. It further investigates how a tangent line captures the shape of a curve near the point of tangency, how to deduce rates of change that cannot be measured from rates of change that are already known, and how to find a function when only its first derivative and its value at a single point are known. | | |
| **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology. | | |
| **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| F-IF 2. | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | |
| F-IF 3. | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* | |
| F-IF 4. | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★ | |
| F-IF 5. | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ | |
| F-IF 6. | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ | |
| F-IF 8. | Write a function defined by an expression in different but equivalent  forms to reveal and explain different properties of the function.  a. Use the process of factoring and completing the square in a  quadratic function to show zeros, extreme values, and symmetry  of the graph, and interpret these in terms of a context.  b. Use the properties of exponents to interpret expressions for  exponential functions. *For example, identify percent rate of change*  *in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and cassify them as representing exponential growth or decay.* | |
| F-IF 9. | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | |
| **Unit Essential Questions**   * What is the importance of maximum and minimum values in real world situations? * What is the relationship between function values and rates of change? * What is the relationship between a function’s rate of change and its optimum value? * When do critical numbers yield local extrema? Similarly, when do hypercritical numbers yield inflection points? | | **Unit Enduring Understandings**  *Students will understand that…*   * There exists a relationship between a function’s maxima and the rate of change at those points, as applied to real world situations. |
| **Unit Objectives**  *Students will know…*   * The Mean value Theorem guarantees that functions increase/decrease depending on the value of the derivative. * The first and second derivatives indicate the behavior of the original function. | | **Unit Objectives**  *Students will be able to…*   * Use the Mean Value Theorem to determine the point at which the derivative equals the average rate of change. * Use the first and second derivative tests to determine maxima, points of inflection, intervals of increase/decrease, and intervals of concavity. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | |
| **Formative Assessments** | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share | * DO-NOW * Notebook * Writing prompts * Exit passes * Self-assessment |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes * Presentations * Unit Projects * Mid-Term and Final Exams | |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | |
| **Teacher Notes:** | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area:** Mathematics **Grade:** High School | | |
| **Unit Title:** Anti-Derivatives and the Definite Integral | | |
| **Domain:** Number & Quantity/Algebra/Functions/Modeling/Geometry | | |
| **Unit Summary:**  The students will evaluate the anti-derivative of a function in general terms and with initial conditions. The students will learn and apply the Fundamental Theorem of Calculus. The students will understand the inverse relationship between a derivative and anti-derivative. | | |
| **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology. | | |
| **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| N-Q.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | |
| N-Q.2 | Define appropriate quantities for the purpose of descriptive modeling. | |
| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |
| A-CED.1 | Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | |
| A-CED.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | |
| A-REI.11 | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | |
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| F-IF.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the* *function is increasing, decreasing, positive, or negative; relative maximums* *and minimums; symmetries; end behavior; and periodicity.* | |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* *h(n) gives the number of person-hours it takes to assemble n engines in a* *factory, then the positive integers would be an appropriate domain for the* *function.* | |
| F-IF.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | |
| F-IF.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | |
| F-BF.1 | Write a function that describes a relationship between two quantities. | |
| F-LE.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | |
| F-LE.4 | For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. | |
| F-LE.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |
| G-MG.1 | Use geometric shapes, their measures, and their properties to describe objects | |
| G-MG.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |
| **Unit Essential Questions**   * How much has a quantity changed? * What is the relationship between a derivative and an integral? * What is the relationship between area under a curve and an integral? | | **Unit Enduring Understandings**  *Students will understand that…*   * A function can be interpreted graphically, analytically and numerically. * The amount that a function has changed over a given time can be interpreted from a graph. * The connection between the definite integral and the area under a curve. |
| **Unit Objectives**  *Students will know…*   * The integral gives the area under the curve. * The rules for anti-differentiation. * The area under a curve is the amount of change of a function over time. | | **Unit Objectives**  *Students will be able to…*   * Evaluate an integral, definite or indefinite. * Apply the Fundamental Theorem of Calculus * Use integration by substitution and integration by parts as needed to evaluate integrals. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | |
| **Formative Assessments** | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share |  |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes * Presentations * Unit Projects * Mid-Term and Final Exams | |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | |
| **Teacher Notes:** | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area:** Mathematics  **Grade:** High School | | |
| **Unit Title:** Differential Equations | | |
| **Domain:** Number & Quantity/Algebra/Functions/Modeling/Geometry | | |
| **Unit Summary:**  The students will solve differential equations using separation of variables. Students will solve differential equations involving exponential growth and decay. Students use various pre-requisite skills and techniques to solve differential equations. | | |
| **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology. | | |
| **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| N-RN.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | |
| N-Q.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | |
| N-Q.2 | Define appropriate quantities for the purpose of descriptive modeling. | |
| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |
| A-CED.1 | Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | |
| A-CED.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | |
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| A-REI.11 | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ | |
| F-IF.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the* *function is increasing, decreasing, positive, or negative; relative maximums* *and minimums; symmetries; end behavior; and periodicity.* | |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* *h(n) gives the number of person-hours it takes to assemble n engines in a* *factory, then the positive integers would be an appropriate domain for the* *function.* | |
| F-IF.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | |
| F-BF.1 | Write a function that describes a relationship between two quantities. | |
| F-LE.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | |
| F-LE.4 | For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. | |
| FL-E.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |
| G-MG.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ | |
| **Unit Essential Questions**   * How much has a quantity changed? * What does a slope field represent? * What techniques of integration can be used to solve a differential equation? | | **Unit Enduring Understandings**  *Students will understand that…*   * Exponential quantities change in a proportional matter. * Differential equations have always been a prime motivation for the study of calculus and remain so to this day * Anti-differentiation techniques are essential for applying the results of calculus |
| **Unit Objectives**  *Students will know…*   * Slope fields represent the numeric derivative of a function at any point * There are various techniques needed for integration. | | **Unit Objectives**  *Students will be able to…*   * Sketch a slope field * Match a differential equation with its slope field. * Solve a separable differential equation using substitution and partial fraction decomposition. * Use Euler’s Method to solve an initial value differential value at a given point. |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | | |
| **Formative Assessments** | | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share | * DO-NOW * Notebook * Writing prompts * Exit passes * Self-assessment | |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes | | * Presentations * Unit Projects * Mid-Term and Final Exams |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | | |
| **Teacher Notes:** | | |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Unit Overview** | | |
| **Content Area:** Mathematics  **Grade:** High School | | |
| **Unit Title:** Applications of Integration | | |
| **Domain:**  Number and Quantity, Algebra, Functions, Modeling, Geometry | | |
| **Unit Summary:**  The students will be able to analyze and solve problems that involve determining how much a quantity changes through the use of anti-derivatives and integration. The connection between the derivative and the integral will be utilized in practical applications involving net change, areas, volumes, etc. | | |
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| **Primary interdisciplinary connections:** Infused within the unit are connections to the 2014 NJCCCS for Mathematics, Language Arts Literacy, Science and Technology. | | |
| **21st century themes:** The unit will integrate the 21st Century Life and Career standards:  CRP2. Apply appropriate academic and technical skills.  CRP4. Communicate clearly and effectively and with reason  CRP6. Demonstrate creativity and innovation.  CRP7. Employ valid and reliable research strategies.  CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11. Use technology to enhance productivity. | | |
| **Learning Targets** | | |
| **Content Standards** | | |
| **Number** | **Common Core Standard for Mastery** | |
| N-Q.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays | |
| N-Q.2 | Define appropriate quantities for the purpose of descriptive modeling. | |
| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ | |
| A-CED.1 | Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | |
| A-CED.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | |
| F-TF.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ | |
| F-TF.7. | (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ | |
| F-TF.7 | (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ | |
| G-SRT.11 | (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | |
| G-MG.3 | .Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ | |
| F-BF.1 | Write a function that describes a relationship between two quantities. | |
| F-LE.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions.   * + Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.   + Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.   + Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | |
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| **Unit Essential Questions**   * How is the velocity integral related to the displacement and/or total distance? * Can bounded area be determined in multiple ways? * Can you determine from a graph which integration strategy would be most efficient in finding a bounded area? * Can you find the volume of a solid of revolution using the methods of discs, washers or shells? * Can you find the volume of solids from known cross sections? | | **Unit Enduring Understandings**  *Students will understand that…*   * How do you calculate a volume of a solid? * How do you calculate area of irregular shapes? * How can you find net change if you know rate of change? |
| **Unit Objectives**  *Students will know…*   * How to find displacement and/or total distance from a given velocity. * How to find boundaries for integrating between intersecting curves. * When to use horizontal or vertical rectangles for finding area based upon given conditions. * Compute the areas and volumes manually and through the use of available technology. | | **Unit Objectives**  *Students will be able to…*   * Find displacement an object at a given moment. * Find the total distance traveled over an given interval of time. * Use algebraic methods or available technology to find point(s) of intersection that will be the boundaries of bounded regions. * Use algebraic methods or available technology to find areas and volumes. * Use dimensional analysis to determine what quantities and units of measure are calculated. *For example – the calculated area under velocity curve (ft/sec) for a given time interval (in sec.) would result in how far (ft.) the object travels.* |

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| **OCEAN COUNTY MATHEMATICS CURRICULUM**  **Evidence of Learning** | |
| **Formative Assessments** | |
| * Observation * Homework * Class participation * Whiteboards/communicators * Think-Pair-Share |  |
| **Summative Assessments**   * Chapter/Unit Test * Quizzes * Presentations * Unit Projects * Mid-Term and Final Exams | |
| **Modifications (ELLs, Special Education, Gifted and Talented)**   * Teacher tutoring * Peer tutoring * Cooperative learning groups * Modified assignments * Alternative assessments * Group investigation * Differentiated instruction * Native language texts and native language to English dictionary * **Follow all IEP modifications/504 plan** | |
| **Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:**  For further clarification refer to NJ Class Standard Introductions at [**www.njcccs.org**](http://www.njcccs.org).   * Graphing Calculator * Microsoft Excel/PowerPoint * Teacher-made tests, worksheets, warm-ups, and quizzes * Computer software to support unit * Smart board * Document camera * [www.ixl.com](http://www.ixl.com) * [www.purplemath.com](http://www.purplemath.com) * [www.Kutasoftware.com](http://www.kutasoftware.com) * [www.Khanacademy.com](http://www.khanacademy.com) * [www.brightstorm.com](http://www.brightstorm.com) * [www.coolmath.com](http://www.coolmath.com) * [www.desmos.com](http://www.desmos.com) | |
| **Teacher Notes:** | |

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| **Common Core State Standards for Mathematics (High School)** | | | | | |
| **Progression of Standards** | | | | | |
|  | **Algebra I** | **Geometry** | **Algebra II** | **Pre Calculus** | **Calculus** |
| **Number & Quantity** |  |  |  |  |  |
| ***The Real Number System (N-RN)*** |  |  |  |  |  |
| Extend the properties of exponents to rational exponents | I | D | M |  |  |
| Use properties of rational and irrational numbers | I | D | M |  |  |
| ***Quantities (N-Q)*** |  |  |  |  |  |
| Reason quanitatively and use units to solve problems | I | D | M |  |  |
| ***The Complex Number System (N-CN)*** |  |  |  |  |  |
| Perform arithmetic operations with complex numbers |  | I | D | M |  |
| Represent complex numbers and their operations on the complex plane |  |  | I | D | M |
| Use complex numbers in polynomial identities and equations |  |  | I | D | M |
| ***Vector and Matrix Quantities (N-VM)*** |  |  |  |  |  |
| Represent and model with vector quantities |  | I |  | D | M |
| Perform operations on vectors |  | I | D | M |  |
| Perform operations on matrices and use matrices in applications | I |  | D | M |  |
| **Algebra** |  |  |  |  |  |
| ***Seeing Structure in Expressions (A-SSE)*** |  |  |  |  |  |
| Interpret the structure of expressions | I | D | M |  |  |
| Write expressions in equivalent forms to solve problems | I | D | M |  |  |
| ***Arithmetic with Polynomials and Rational Expressions (A-APR)*** |  |  |  |  |  |
| Perform arithmetic operations on polynomials | I | D | M |  |  |
| Understand the relationship between zeros and factors of polynomials | I |  | D | M |  |
| Use polynomial identities to solve problems | I |  | D | M |  |
| Rewrite rational expressions | I | D | M |  |  |
| ***Creating Equations (A-CED)*** |  |  |  |  |  |
| Create equations that describe numbers or relationships | I | D | M |  |  |
| ***Reasoning with Equations and Inequalities (A-REI)*** |  |  |  |  |  |
| Understand solving equations as a process of reasoning and explain the reasoning | I | D | M |  |  |
| Solve equations and inequalities in one variable | I | D | M |  |  |
| Solve systems of equations | I |  | D | M |  |
| Represent and solve equations and inequalities graphicallly | I |  | D | M |  |
| **Functions** |  |  |  |  |  |
| ***Interpreting Functions (F-IF)*** |  |  |  |  |  |
| Understand the concept of a function and use function notation | I | D | M |  |  |
| Interpret functions that arise in applications in terms of the context | I | D | M |  |  |
| Analyze functions using different representations |  |  |  |  |  |
| ***Building Functions (F-BF)*** | I |  | D | M |  |
| Build a function that models a relationship between two quantities | I | D | M |  |  |
| Build new functions from existing functions | I |  | D | M |  |
| ***Linear, Quadratic, and Exponential Models (F-LE)*** |  |  |  |  |  |
| Construct and compare linear, quadratic, and exponential models and solve problems | I |  | D | M |  |
| Interpret expressions for functions in terms of the situation they model | I |  | D | M |  |
| ***Trigonometric Functions (F-TF)*** |  |  |  |  |  |
| Extend the domain of trigonometric functions using the unit circle |  | I | D | M |  |
| Model periodic phenomena with trigonometric function |  | I | D | M |  |
| Prove and apply trigonometric identities |  | I |  | D | M |
| **Geometry** |  |  |  |  |  |
| ***Congruence (G-CO)*** |  |  |  |  |  |
| Experiment with transformations in the plane |  | I |  | D | M |
| Understand congruence in terms of rigid motions |  | I |  | D | M |
| Prove geometric theorems |  | I |  | D | M |
| Make geometric constructions |  | I |  | D | M |
| ***Similarity, Right Triangles, and Trigonometry (G-SRT)*** |  |  |  |  |  |
| Understand similarity in terms of similarity transformations |  | I |  | D | M |
| Prove theorems involving similarity |  | I |  | D | M |
| Define trigonometric ratios and solve problems involving right triangles | I | D |  | M |  |
| Apply trigonometry to general triangles |  | I |  | D | M |
| ***Circles (G-C)*** |  |  |  |  |  |
| Understand an apply theroems about circles |  | I |  | D | M |
| Find arc lenghts and areas of sectors of circles |  | I |  | D | M |
| ***Expressing Geometric Properties with Equations (G-GPE)*** |  |  |  |  |  |
| Translate between the geometric description and the equation for a conic section |  | I |  | D | M |
| Use coordinates to prove simple geometric theorems algebraically |  | I |  | D | M |
| ***Geometric Measurement and Dimension (GGMD)*** |  |  |  |  |  |
| Explain volume formulas and use them to solve problems |  | I |  | D | M |
| Visualize relationships between two-dimensional and three-dimensional objects |  | I |  | D | M |
| ***Modeling With Geometry (G-MG)*** |  |  |  |  |  |
| Apply geometric concepts in modeling situations |  | I |  | D | M |
| **Statistics and Probability** |  |  |  |  |  |
| ***Interpreting Categorical and Quantative Data S-ID)*** |  |  |  |  |  |
| Summarize, represent, and interpret data on a single count or measurement variable | I |  | D | M |  |
| Summarize, represent, and interpret data on two categorical and quantitative variables | I |  | D | M |  |
| Interpret linear models | I |  | D | M |  |
| ***Making Inferences and Justifying Conclusions (S-IC)*** | I |  | D | M |  |
| Understand and evaluate random processes underlying statistical experiments | I |  | D | M |  |
| Make inferences and justify conclusions from sample surveys, experiments and observational studies | I |  | D | M |  |
| ***Conditional Probability and the Rules of Probability S-CP)*** |  |  |  |  |  |
| Understand independence and conditional probability and use them to interpret data | I |  | D | M |  |
| Use the rules of probability to compute probabilities of compound events in a uniform probability model | I |  | D | M |  |
| ***Using Probability to Make Decisions (S-MD)*** |  |  |  |  |  |
| Calculate expected values and use them to solve problems | I |  | D | M |  |
| Use probability to evaluate outcomes of decisions | I |  | D | M |  |